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THE PHYSICAL MEANING OF THE STAR-MAGNITUDE

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THE PHYSICAL MEANING OF THE STAR-MAGNITUDE.

By R. de Kövesligethy.

By assuming Pogson's numerical value for the constant of the Fechner psycho-physical law, we get the definition of the star-magnitude in the form

$$m - m_0 = -2.5 \log \frac{J}{J_0} \qquad (1)$$

which, for stars equal to or fainter than the sixth magnitude, represents tolerably well the connection between the measured intensity J and J_0 and the estimated magnitude m, m_0 of any two stars. This definition, however, involves merely subjective physiological elements, inasmuch as the visual intensity depends not only upon the objective continuous spectrum of the star, but varies also with the limiting wave-lengths of perception and with the sensibility of the eye, which is itself a function of the wave-length. Thus even for another suitable choice of the Pogson constant the differences of magnitude seem altered when the perceiving medium of the spectrum is changed; *e. g.*, when the photographic film is used instead of the normal eye.

The question of a physical definition of the star magnitude is therefore still open, though in many cases it might be of considerable value, especially for the estimation of processes operating in new stars. Of course I do not here allude to the fact sometimes invoked in investigations of the dimensions of the stellar system, that stars of equal intensity seem fainter by *one* magnitude if removed to a 1.585-fold distance, but I seek the changes in the physical condition of a star whose brightness has varied by *one* magnitude.

Let

$$i = \Lambda f(\lambda, \mu) \qquad (2)$$

be the intensity of a continuous spectrum between the wave-lengths λ and $\lambda + d\lambda$. Then the visual intensity becomes

$$I = \Lambda \int_{\lambda_1}^{\lambda_2} s f(\lambda, \mu)\, d\lambda, \qquad (3)$$

where the sensibility of the eye, s, is a function of the wave-length, while λ_1 and λ_2 denote the limits of the visual spectrum. The function $\Lambda f(\lambda, \mu)$ contains at least two independent parameters, as I have shown elsewhere[1] (since it would lead to an equation of dispersion without any constant). Let Λ be the total intensity of the spectrum and μ the wave-length of the maximum of intensity. Thus we have

$$\int_0^\infty f(\lambda, \mu)\, d\lambda = i\,.$$

The analytical form of the sensibility function s is quite unknown (for geometrical representation, see this JOURNAL, 11, 18–22). Numerically it might be determined by comparisons of the visual intensity with energy observations made with an absolutely black bolometer. Its differential equations could be measured in a simpler way by the probable error with which a homogeneous light-field of given wave-length can be inserted on the appropriate place of a continuous spectrum. Here we can get on without the explicit expression of the sensibility, inasmuch as a mean value of it between the limits of the spectrum may be placed as a factor before the integral. Thus the visual intensity becomes

$$I = \sigma \Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda\,, \tag{4}$$

and σ remains constant for all stars of the same spectral composition, that is of the same type, while it varies from type to type.

This variation is very little, however, and, beyond the limits of exactness of photometric measures, may be neglected in practice. Indeed all photometric measures, even of differently colored stars, are usually made by comparison with a few standard stars, and likewise the same color of comparison star was always used in the *Potsdamer Durchmusterung*, being about equal to the mean color of the second stellar type.

[1] *Monthly Notices*, 58, No. 3, 115; *Grundzüge einer theoretischen Spektralanalyse* Halle a/S. 1890, p. 151.

Thus we become—at least in practice—independent of the unknown coefficient σ, and the Pogson formula gives for the same star whose magnitude changed by *one* order $(m = m_0 + 1)$:

$$1 = -2.5 \log \frac{\Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda}{\Lambda_0 \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu_0)\, d\lambda}, \tag{5}$$

as conditional equation for the changes necessary in Λ_0 and μ_0 to produce such effect. Taking into consideration what we have said of the sensibility factor, this equation holds good, even if the change of light was accompanied by a variation of the color amounting to an interval of two types.

The two parameters of the spectrum, Λ and μ, are functions of the temperature and the density,[1] and, though known functions, do not allow a uniform solution of the above equation. The manner in which the temperature and density changed remains in every case quite arbitrary, and thus a uniform physical definition of the star magnitude cannot be given.

The question becomes resolvable in a uniform manner, however, if we assume that the radiating body is an absolutely black one. Inasmuch as the superficial layer, which produces the greatest part of the continuous spectrum, may be regarded thick enough, our hypothesis holds also good for the fixed stars.

Now we know two analytical expressions for the continuous spectrum; the one generally applicable for any body is by the author:[2]

$$i = \frac{4}{\pi} \mu \Lambda \frac{\lambda^2}{(\lambda^2 + \mu^2)^2}; \tag{6}$$

the other, which refers only to absolutely black bodies, is that of Wien-Paschen,[3] and may be written in the form

$$i = \frac{5^4 \mu^4}{6} \Lambda \lambda^{-5} e^{-\frac{5\mu}{\lambda}}. \tag{6'}$$

[1] "Ueber die beiden Parametergleichungen der Spektralanalyse," *Math. u. Naturw. Ber. aus Ungarn*, 16, 1–49.

[2] *Grundzüge e. theor. Spektralanalyse*, p. 157.

[3] This JOURNAL, 10, 40.

For an absolutely black body, Λ obeys the Stefan law of radiation, while μ is given by the Wien formula. If θ denotes the absolute temperature, then

$$\frac{\Lambda}{\Lambda_0}=\frac{\theta^4}{\theta_0^4}\,,\ \text{and}\ \mu\theta=2880\ , \tag{7}$$

provided that μ is given in thousandth parts of the millimeter. It may be said, in brief, that I was familiar with this ten years before the publication of Wien's *Verschiebungsgesetz*, and that I determined its constant provisionally by the observations of G. Müller[1] on the absorption of the air, in a most troublesome way, as 2088. Similarly I used this equation and the value of the constant for the determination of the temperature of the Sun's chromosphere. It gave 1800°, or, with the present value of the constant, 2475° of the absolute scale, being 1.1631 the wave-length of the maximum intensity of an absolutely black body having the same temperature as the chromosphere.[2]

Putting now

$$F_1(\mu)=\frac{4}{\pi}\mu\int_{\lambda_1}^{\lambda_2}\frac{\lambda^2}{(\lambda^2+\mu^2)^2}d\lambda$$

$$=\frac{2}{\pi}\left[\text{arc tang}\,\frac{\mu(\lambda_2-\lambda_1)}{\mu^2+\lambda_1\lambda_2}-\mu\,\frac{(\lambda_2-\lambda_1)(\mu^2-\lambda_1\lambda_2)}{(\lambda_1^2+\mu^2)(\lambda_2^2+\mu^2)}\right]\,, \tag{8}$$

and

$$\frac{\lambda_1}{\mu}=\text{tang}\ \phi_1\ ;\ \frac{\lambda_2}{\mu}=\text{tang}\ \phi_2\ ,$$

we get in a more simple form

$$F_1(\mu)=\frac{2}{\pi}\Big[(\phi_2-\phi_1)-\sin(\phi_2-\phi_1)\cos(\phi_2+\phi_1)\Big]\ ,$$

and similarly in the other equation

$$F_2(\mu)=\frac{5^4\mu^4}{6}\int_{\lambda_1}^{\lambda_2}\lambda^{-5}e^{-\frac{5\mu}{\lambda}}\,d\lambda$$

$$=\frac{125}{6}\left\{e^{-\frac{5\mu}{\lambda_2}}\left(\frac{\mu^3}{\lambda_2^3}+\frac{3\mu^2}{5\lambda_2^2}+\frac{6\mu}{25\lambda_2}+\frac{6}{125}\right)\right.$$

$$\left.-e^{-\frac{5\mu}{\lambda_1}}\left(\frac{\mu^3}{\lambda_1^3}+\frac{3\mu^2}{5\lambda_1^2}+\frac{6\mu}{25\lambda_1}+\frac{6}{125}\right)\right\}\,. \tag{8'}$$

[1] *Astr. Nachr.*, 103, 241, 1882.

[2] *Grundzuge*, p. 185.

we have generally

$$I = \Lambda F(\mu) \ , \tag{9}$$

and thus the Pogson equation (5) becomes

$$\log \mu - \frac{1}{4} \log F(\mu) = 0.1 + \log \mu_0 - \frac{1}{4} \log F(\mu_0) \ . \tag{10}$$

Thus if we know the average values of μ_0 for the individual stellar types we may calculate the variations in μ necessary to produce a change from *one* magnitude in the luminosity of the star. Early spectral-photometric observations of thirty-four fixed stars[1] led me to the following values, corrected for the effect of the atmosphere:

Type of the star:		I.	II.	III.	
Average μ_0	=	0.45	0.53	0.60,	(11)

which give, according to the two hypotheses made for $F_1(\mu)$ and $F_2(\mu)$, the limits of the visual spectrum being assumed at $\lambda_1 = 0.39$ and $\lambda_2 = 0.76$, the equations

I. type;	$\log \mu - \frac{1}{4} \log F_1(\mu) + 0.0923 = 0$	and $\log \mu - \frac{1}{4} \log F_2(\mu) + 0.1679 = 0$	
II.	" " $+0.0082 = 0$	" " $+0.0881 = 0$	(12)
III.	" " $-0.0592 = 0$	" " $+0.0200 = 0$	

The solutions for the three types are, accordingly,

I. type	$\mu = 0.546$	and $\mu = 0.551$	
II.	0.636	0.633	(13)
III.	0.715	0.706,	

where the first values refer to the first hypothesis for $F_1(\mu)$, the other column to the second hypothesis $F_2(\mu)$.

For facilitating the solution of (10) we give here a short numerical table of the function $F_1(\mu)$ and $F_2(\mu)$.

μ	Log $F_1(\mu)$	Log $F_2(\mu)$	μ	Log $F_1(\mu)$	Log $F_2(\mu)$
0.40	9.4073	9.6827	0.65	9.2354	9.5412
45	3821	6842	70	1931	4818
50	3507	6671	75	1499	4162
55	3150	6357	80	9.1066	9.3454
60	9.2762	9.5930			

The part of the energy falling in the visual spectrum is considerably greater if the spectrum is assumed in the exponential form instead of the algebraic form. But in both cases the

[1] *Beobacht. angest. am astrophys. Observ. O-Gyalla*, 9, 21–41, 1888.

intensity vanishes for $\mu = 0$ and $\mu = \infty$, and reaches a maximum value for $\mu = 0.309$ and $\mu = 0.429$ respectively. These values, therefore, correspond to the impression of the purest white in the mixed light of the source. I remark that the gradual increasing and the subsequent decreasing of the intensity with increasing temperature is proved, at least in single cases, by observations of Lucas.

In virtue of the second part of equation (7), the solutions (13) correspond to the following temperatures:

Type.	θ_0	θ_1	θ_2	$\frac{\theta_0}{\theta_1}$	$\frac{\theta_0}{\theta_2}$	
I.	6400°	5274°	5230°	1.213	1.224	
II.	5434	4527	4547	1.200	1.195	(14)
III.	4799	4025	4080	1.191	1.176	
			Mean of temperature ratio:	1.201	1.198.	

Therefore, if a white star changes in magnitude by *one* order, it is the same as if its superficial temperature fell from 6400° to 5274° or to 5230° respectively. The temperatures in the two hypotheses are almost identical, and the ratio of them is also sensibly the same, but shows a slight variation from type to type. If we therefore assume on the two hypotheses that a variation of *one* magnitude corresponds to an increase of temperature in the ratio of 1 : 1.201 or of 1 : 198, the equation (1) may be written in this form:

$$m - m_0 = -12.58 \log \frac{\theta}{\theta_0}, \text{and } m - m_0 = -12.74 \log \frac{\theta}{\theta_0}, \quad (15)$$

and the assumption of an average ratio gives for the stars of the first and third type errors of 0.056 and 0.044, of 0.118 and 0.103 mag., respectively, which on my hypothesis do not reach, on the other scarcely exceed, the limits of exactness of photometric measures.

We have, then, the following theorem:

When the light of a star in form of a point, of nearly absolute black character, increases by one magnitude, then, independently of the color of its light, its superficial temperature has lessened by 20 per cent.

We may further remark, that the "superficial" temperatures (*i. e.*, temperature of the layer, which produces the principal part of the continuous spectrum) given in (14) under θ_0 represent lower limits. For if any body and an absolutely black body have the same wave-length of intensity maximum, the latter is always of lower temperature.

The above investigation may be of some use in its application to new stars. The star *T Coronae* increased by three magnitudes in about two and one half hours; that is to say, during this time its temperature was elevated about 0°.5 per second, a most probable quantity, whether we assign the apparition to a collision with a cosmic cloud or to an eruption of the still fluid inner magma.

O-Gyalla Observatory,
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